Ch 1. Review

I. Units and Dimensions

Units - Standardized values from which we make measurements

Metric means power of 10
So the metric system is a power of 10 system.

3 major systems of units
There are 7 fundamental units

SI (System International) *MKS (meter, kilogram, second)*	*British*	*CGS (centimeter, gram, sec.)*
1. Time - second (s)	1. Time - second (s)	1. Time - second (s)
2. Mass - kilograms (kg)	2. Mass – slug	2. Mass - grams (g)
3. Length - meters (m)	3. Length - foot (ft.)	3. Length - centimeters (cm)
4. Temp. - Kelvin (K)	4. Temp. - Fahrenheit (F)	4. Temperature - Kelvin (K)
5. Current - Ampere (A)	5. Current - Ampere (A)	5. Current - Ampere (A)
6. Amount - mole (mol)	6. Amount – mole (mol)	6. Amount - mole (mol)
7. Luminous Intensity - Candela (cd)	7. Luminous Intensity - ?	7. Luminous Intensity - ?

Dimensions

Example: Consider a rectangle with sides 2m and 3m

The Area, $A = 2\text{m} \bullet (3\text{m}) = 6\text{m}^2$

The *dimensions* for area is length squared.

The *units* in this case, is meters squared.

Non geometric quantities can also have dimensions.

Example. Consider the speed of a car that travels 10 miles in 2 hrs.

The Speed, $v = \dfrac{10\text{ mi.}}{2\text{ hrs}} = 5\ \dfrac{\text{mi.}}{\text{hr}}$

The *dimensions* for speed is length divided by time.

The *units* for speed in this case, are miles per hour.

II. Conversion of Units

Factor -Label Method (Dimensional Analysis)

Conversion Factors - Usually found in front/back cover or appendix.

Things Like: 1 mi. = 1610 m
1 hr = 60 min.
1 min. = 60 sec.

Example Questions:

Convert 37.25 $\frac{\text{mi.}}{\text{hr}}$ to $\frac{\text{cm}}{\text{sec.}}$

$$\frac{37.25\,\text{mi}}{\text{hr}} \times \frac{1\,\text{hr}}{3600\,\text{sec}} \times \frac{1610\,\text{m}}{1\,\text{mi}} \times \frac{100\,\text{cm}}{1\,\text{m}} = \boxed{1665.903\ \text{cm/s}}$$

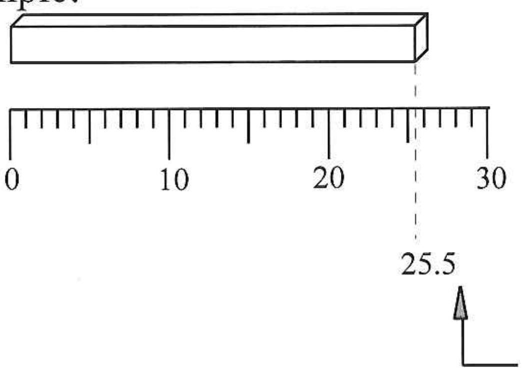
Convert 10 $\frac{\text{mi.}}{\text{hr}^2}$ to $\frac{\text{cm}}{\text{sec.}^2}$

$$\frac{10\,\text{mi}}{\text{hr} \times \text{hr}} \times \frac{1\,\text{hr}}{3600\,\text{s}} \times \frac{1\,\text{hr}}{3600\,\text{s}} \times \frac{1610\,\text{m}}{1\,\text{mi}} \times \frac{100\,\text{cm}}{1\,\text{m}} = \boxed{.124\ \text{cm/s}^2}$$

III. Significant Figures

The number of significant figures in any given number includes all of the digits that are of definite value plus the first digit to the right with an uncertain value.

Example:



The last digit in this number is uncertain (estimated) but it is still significant and should be recorded.

Rules for significant Figures

Rule	Example	# of Sig. Figs.
1. Nonzero digits are always significant	32,526	5
2. All final zeros after the decimal point are significant	36.00	4
3. Zeros between two other significant digits are always significant	6.0002	5
4. Zeros used solely for spacing the decimal point are not significant	0.003	1

Note:

FOR ***multiplying or dividing*** with significant figures, a ***solution*** can never have more significant figures than the least number of sig. figs. in any component number within the calculation.

FOR ***addition and subtraction*** a solution may can never be more precise than the least precise measurement.

Rules for Significant Figures - Explanation by Example

Significant Digits—Division

Divide 36.5 m by 3.414 s.

Solution:

$$\frac{36.5\ m}{3.414\ s} = 10.69\ m/s$$ This is correctly stated as 10.7 m/s.

Significant Digits—Addition and Subtraction

Add 24.686 m + 2.343 m + 3.21 m.

Solution:

```
  24.686 m
   2.343 m
   3.21  m
  30.239 m
= 30.24  m
```

Note that 3.21 m is the least precise measurement. Round off the result to the nearest hundredth of a meter. Follow the same rules for subtraction.

Significant Digits—Multiplication

Multiply 3.22 cm by 2.1 cm.

Solution:

```
       3.2 2 cm
         2.1 cm
       3 2 2
     6 4 4
     6.7 6 2 cm²
```

This is correctly stated as 6.8 cm^2.

The less precise factor, 2.1 cm, contains two significant digits. Therefore the product has only two. Note that each circled digit is doubtful, either because it is an estimated measurement or is multiplied by an estimated measurement. Since the 7 in the product is doubtful, the 6 and 2 are certainly not significant. The answer is best stated as 6.8 cm^2.

IV. Exponential Notation (Scientific Notation)

The numerical part of a measurement is expressed as a number between 1 and 10 multiplied by a whole number power of 10.

$$M \times 10^n$$

Whole number

number between 1 and 10

Notation - sometimes the $\times 10$ is replaced by an E

For example: $3 \times 10^4 = 3$ E 4 or $3 \times 10^{-4} = 3$ E-4

Note: Positive sign on the power of 10 means "big" number
Negative sign on the power of 10 means "small" number

$3 \times 10^4 = 30{,}000$ $\qquad$ $3 \times 10^{-4} = 0.0003$

Example: Express 450,000 in exponential notation

4.5×10^5

Example: Express 0.000 000 000 000 000 000 000 000 000 000 911 kg in exponential notation

9.11×10^{-31} kg ← mass of an electron!

Rules for adding, subtracting, multiplying and dividing Exponents

- ☐ Adding and Subtracting

  Must have like exponents

  4.0×10^6 m + 3.0×10^5 m =

  $40 \times 10^5 m + 3 \times 10^5 m = 43 \times 10^5 m = \boxed{4.3 \times 10^6 m}$

  -OR-

  $4 \times 10^6 m + .3 \times 10^6 m = \boxed{4.3 \times 10^6 m}$

  4.0×10^{-6} kg - 3.0×10^{-7} kg =

  $40 \times 10^{-7} kg - 3 \times 10^{-7} kg = 37 \times 10^{-7} kg = \boxed{3.7 \times 10^{-6} kg}$

  -OR-

  $4 \times 10^{-6} kg - .3 \times 10^{-6} kg = \boxed{3.7 \times 10^{-6} kg}$

- ☐ Multiplying

  Add the exponents

  $(3.0 \times 10^6$ m$)$ $(2.0 \times 10^3$ m$)$ = $\boxed{6.0 \times 10^6 m}$

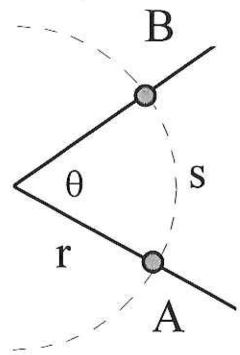
  $(2.0 \times 10^{-5}$ m$)$ $(4.0 \times 10^9$ m$)$ = $\boxed{8.0 \times 10^4 m}$

- ☐ Dividing

  Subtract the exponents

  $$\frac{8 \times 10^6 \text{ kg}}{2 \times 10^{-2} \text{ kg}} = \boxed{4 \times 10^8 kg}$$

V. Review of Trigonometry



r = radius
s = arc length

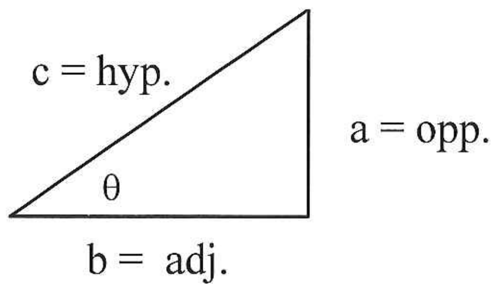
$\vartheta = \frac{s}{r}$ = Angle in radians ⇒ $s = r\theta$

$\boxed{2\pi \text{ radians} = 360^{\circ}}$

Therefore, $\boxed{\frac{\pi}{180^{\circ}} = 1^{\circ}}$

Consider the right triangle

Pythagorean Theorem:

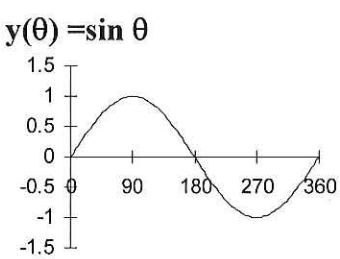


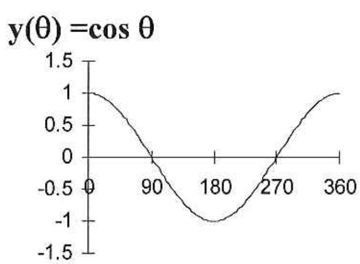
$\boxed{c^2 = a^2 + b^2}$

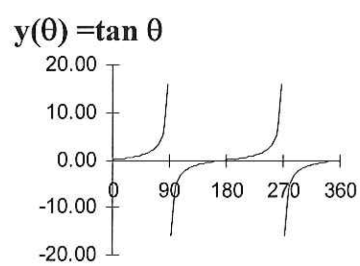
also,

$\sin\theta = \frac{a}{c} = \frac{opp.}{hyp.}$ $\cos\theta = \frac{b}{c} = \frac{adj.}{hyp.}$ $\tan\theta = \frac{a}{b} = \frac{opp.}{adj.}$

$\theta = \sin^{-1}\left(\frac{a}{c}\right)$ $\theta = \cos^{-1}\left(\frac{b}{c}\right)$ $\theta = \tan^{-1}\left(\frac{a}{b}\right)$

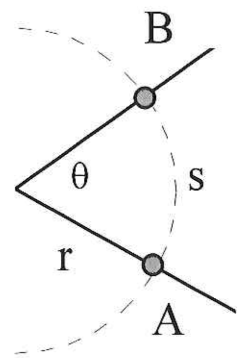






Example Problems (Trigonometry)

1. If a radius of 3m sweeps out 34 degrees what is the magnitude of the arc length?



$34° \times \frac{\pi \text{ rad}}{180°} = .5934 \text{ rad}$

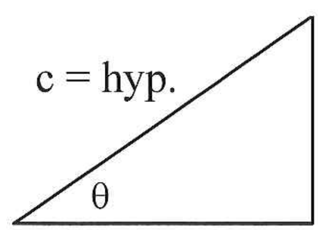
$s = r\theta$

$= 3m(.5934 \text{ rad}) \Rightarrow \boxed{s = 1.78m}$

2. Convert 35 radians to degrees.

$35 \text{ rad} \times \frac{180°}{\pi \text{ rad}} = \boxed{2005.35°}$

3. c = 7 m and a = 3.5 m
   a.) What is b?



$a^2 + b^2 = c^2$

$3.5^2 + b^2 = 7^2$

$\boxed{b = 6.062m}$

   b.) What is θ?

$\sin\theta = \frac{a}{c} = \frac{3.5}{7}$ $\cos\theta = \frac{b}{c} = \frac{6.062}{7}$ $\tan\theta = \frac{a}{b} = \frac{3.5}{6.06}$

$\sin\theta = \frac{1}{2}$ -OR- $\boxed{\theta = 30°}$ -OR- $\boxed{\theta = 30°}$

$\boxed{\theta = 30°}$

VI. Functions - Linear, Quadratic, Inverse

$y(x)$ ⇒ This reads y as a function of x (or just "y of x")

For example $y(x) = 3x + 10$

Independent Variables

for ex: Speed vs time.

As the variable x is changed a new y value is obtained

As time goes on, we measure speed. The speed is dependent on time. We decide what time to record; we're in control of time.

x is the therefore the independent variable

Independent variables are plotted on the x-axis

Note: This is the variable the experimenter changes

Dependent Variables

y is then the dependent variable

Dependent variables are plotted on the y-axis

IMPORTANT:

If you are asked to plot speed as a function of time, that is *speed vs. time*, you would plot speed on the y-axis and time on the x-axis.

Always, $\boxed{\textbf{(y-axis) vs. (x-axis)}}$

Linear Functions

$$y(x) = mx + b$$

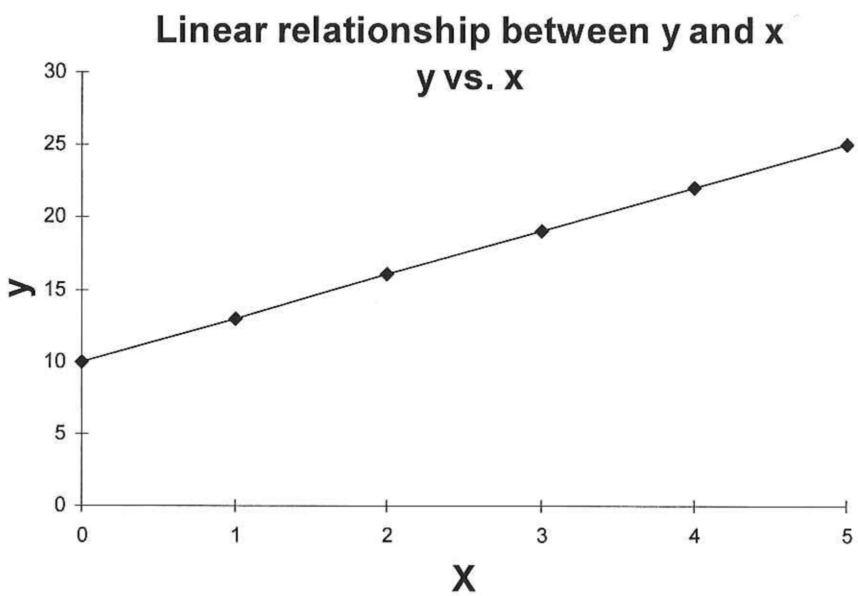
$$m = \text{slope} = \frac{\Delta y}{\Delta x} = \frac{rise}{run}$$

$$b = y\text{ - intercept}$$

Example:

$$y(x) = 3x + 10$$

x	y
0	10
1	13
2	16
3	19
4	22
5	25



$$m = \frac{\Delta y}{\Delta x} = \frac{16-10}{2-0} = 3$$

Any two points should give a slope of 3 since its a linear relationship!

Quadratic Functions

$$y(x) = k\,x^2$$

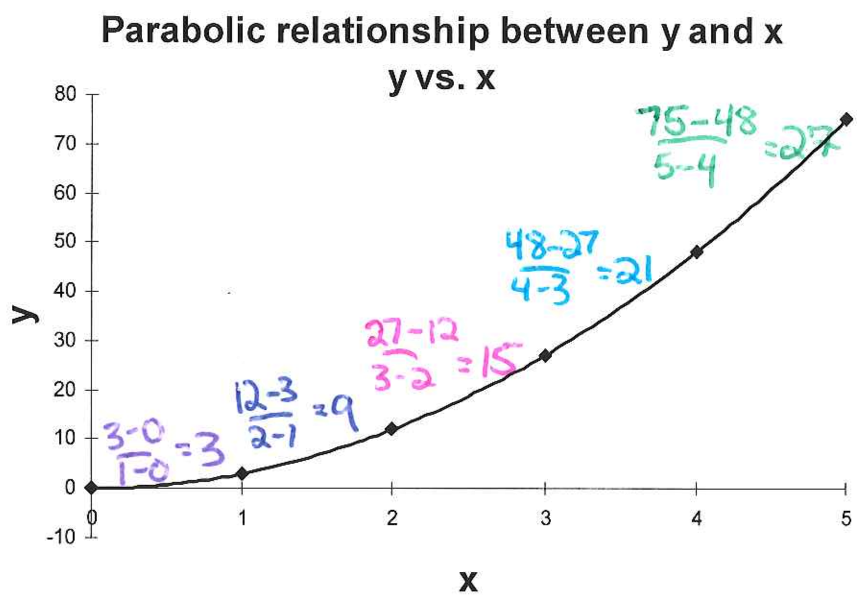
$$k = \text{constant}$$

Slope is <u>not</u> constant as x increases, so does the slope

Example:

$$y(x) = 3x^2$$

x	y
0	0
1	3
2	12
3	27
4	48
5	75



On a related note:

$$ax^2 + bx + c = 0 \quad \Leftarrow \quad \text{Quadratic Equation}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \Leftarrow \quad \text{Solution}$$

* make sure you have this in your calculator. it will be <u>very</u> helpful in later chapters.

Inverse Functions

$$y(x) = k\left(\frac{1}{x}\right) = k\,x^{-1}$$

$$k = \text{constant}$$

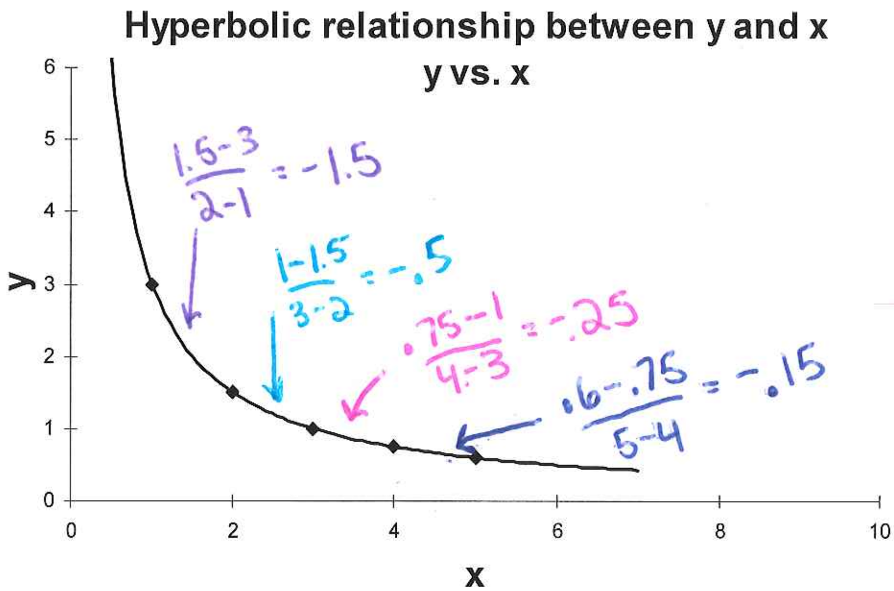
* All slopes are negative

* As x increases, slope gets less negative (or increases)

Example:

$$y(x) = \frac{3}{x}$$

x	y
0	indeterminate
1	3
2	1.5
3	1
4	0.75
5	0.6



Measurement and Uncertainty

Precision - tells about the finest increment that can be measured

For example: The width of a board is 23.2 ± 0.1 cm

The measurement was precise to within a tenth of a cm.

± 0.1 cm is called the *approximate* or *estimated* uncertainty of the measurement.

When a measurement is given without an approximate uncertainty we will assume that the measurement is ± 1 unit of the last specified digit.

Precision in decimal form:

0.1 is what % of 23.2

$0.1 = \_\_\_\_\_\% \times 23.2$

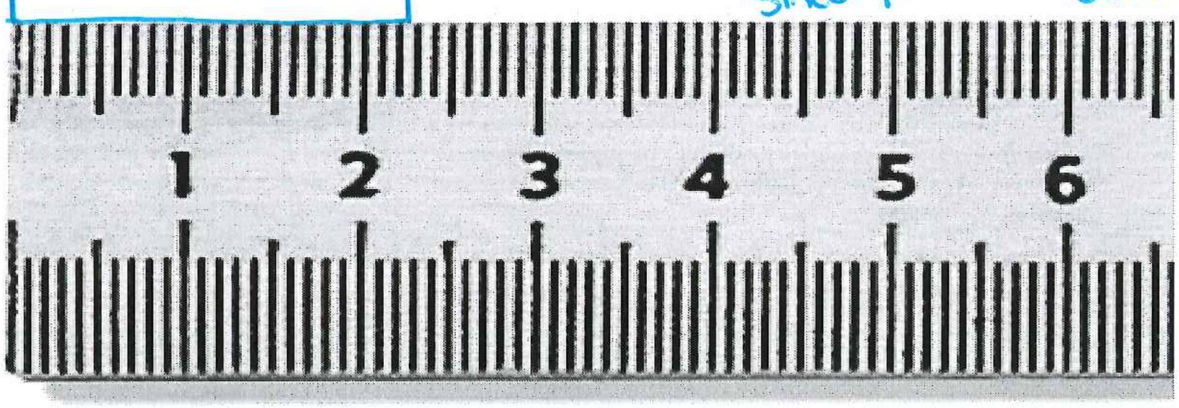
$$\frac{0.1}{23.2} = 0.0043 = 0.004$$

$\therefore$ the % uncertainty is 0.4%

For example, say you are using a meter stick:

random object

You can say this is 2.2 cm, but its ± .1cm since you can't be sure its exactly on the .2 cm mark



Accuracy - tells how well a measurement compares to a standard or theoretical value.

For example:

The current best value for the speed of light in a vacuum is

$2.998 \times 10^8 \frac{m}{s}$.

If a measurement was made and the value $3.001 \times 10^8 \frac{m}{s}$ was obtained then

$$\left| 3.001 \times 10^8 \frac{m}{s} - 2.998 \times 10^8 \frac{m}{s} \right| = 0.003 \times 10^8 \frac{m}{s}$$

and we can say that the measurement was accurate to within

$0.003 \times 10^8 \frac{m}{s}$

Accuracy in percent form:

$0.003 \times 10^8 \frac{m}{s}$ is what % (Decimal) of $2.998 \times 10^8 \frac{m}{s}$

$0.003 \times 10^8 \frac{m}{s} =$ ______ % x $2.998 \times 10^8 \frac{m}{s}$

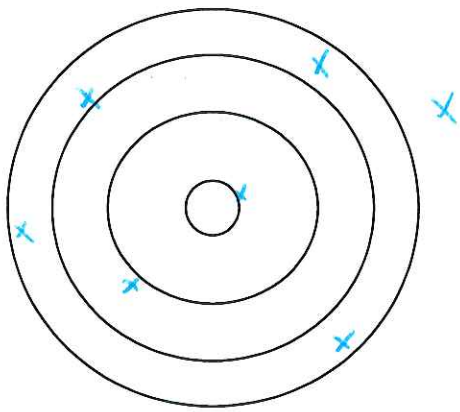
$$\frac{0.003 \times 10^8 \frac{m}{s}}{2.998 \times 10^8 \frac{m}{s}} = 0.001 \qquad \Leftarrow \text{ \% Error in decimal form}$$

The percent error is 0.1%

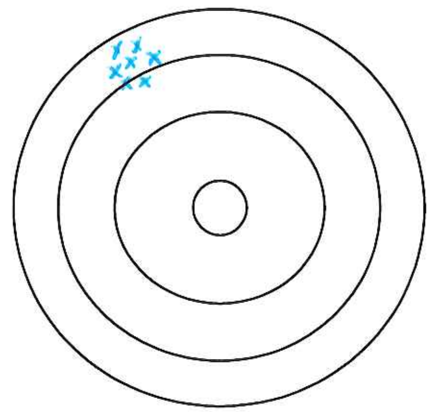
Notice, to get the percent error the following procedure was used

$$\boxed{\frac{|\text{Measured Value - Theoretical Value}|}{\text{Theoretical Value}} \times 100 = \%\text{Error}}$$

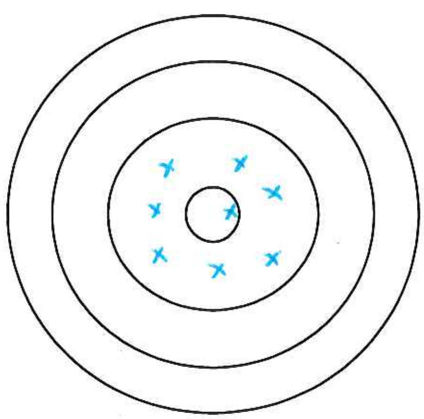
Precision and Accuracy



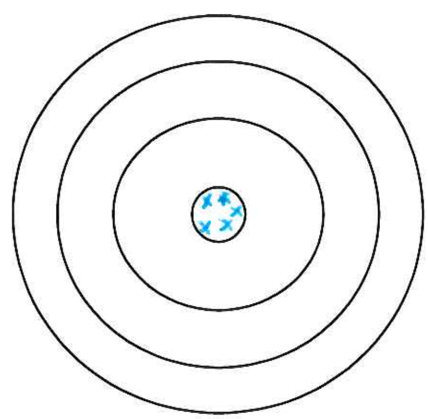
Neither Precise nor accurate



Precise, not accurate

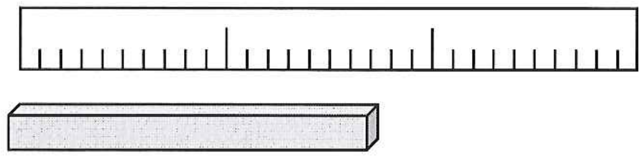


Accurate, not precise

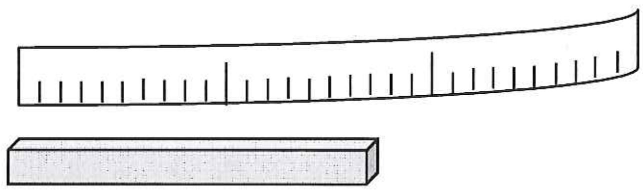


Precise and accurate

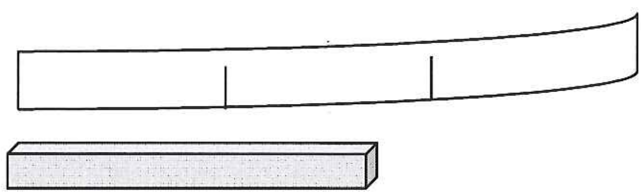
Precision and Accuracy



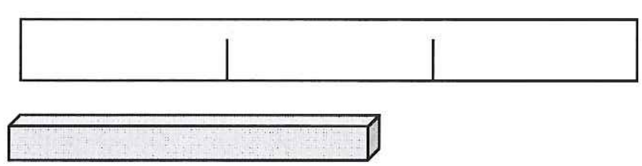
Matches standard in length and can measure to feet and tenths of a foot and estimate to hundredths of a foot.



Does not match the standard in length and can measure to tenths of a foot and estimate to hundredths of a foot.



Does not match the standard in length and can measure to feet and estimate to tenths of feet.



Does match the standard in length and can measure to feet and estimate to tenths of feet.